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Locally Rotationally Symmetric Bianchi Type-V String Cosmological Model for Stiff Perfect Fluid with Cosmological Term Λ

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Abstract

The present paper deals with locally rotationally symmetric Bianchi type-V string cosmological model for stiff perfect fluid with cosmological term Λ. To obtain the explicit solution, we suppose that Λ is inversely proportional to R^3. We have also discussed physical and geometrical characteristics of the cosmological model.

Keywords: LRS Bianchi type-V, massive string, stiff perfect fluid, cosmological term.

Introduction

Bianchi type-V cosmological models are more interesting as these models include particular cases of isotropy and at some instant of cosmic time these models also allow arbitrary small anisotropy levels. These models are the natural generalization of Friedman-Robertson-Walker cosmological models with negative curvature. Bianchi type-V cosmological models have been studied by number of authors viz. Bali [2], Deo et al. [3], Dwivedi et al. [6], Kandalkar et al. [10], Roy et al. [13].

A broad range of observations compellingly recommend that our universe hold a non-zero cosmological constant. Cosmological models with dynamic cosmological term Λ become popular as these models are used to resolve the cosmological constant problem in a natural manner. Various Bianchi type cosmological models with cosmological term Λ have been discussed by various researchers viz. Bali and Tinker [1], Dubey et al. [4], Dwivedi et al. [5], Humad et al. [7, 8], Pawar et al. [12]. Furthermore, the investigations of relativistic theories of gravitation of the cosmological models generally have the energy momentum tensor of matter generated by perfect fluid. Various Bianchi type cosmological models for perfect fluid distribution are considered by Sharma et al. [14], Tripathi et al. [15], Tripathi [16], Tyagi et al. [17, 18].

Non shearing locally rotationally symmetric Bianchi type-III string cosmological model in presence of magnetic flux with bulk viscosity has studied by Patil et al. [11]. Jaiswal et al. [9] have investigated shear free Bianchi type-V string cosmological model in self creation cosmology.

Motivated by aforesaid, in this paper, we deal with LRS Bianchi type-V string cosmological model for stiff perfect fluid with cosmological term Λ. For the complete solution, we assume that Λ is inversely proportional to R^3. We have also discussed physical and geometrical characteristics of the cosmological model.

Metric and Field equation

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The line element for Bianchi type-V space-time is considered as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2\alpha x}(dy^2 + dz^2) \tag{1}$$

where A and B depends on cosmic time t only and α is a non zero constant.

In the geometrized unit (c=8πG=1), the Einstein's field equation is defined as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \tag{2}$$

where R_i^j is Ricci tensor and $R = g^{ij} R_{ij}$ is Ricci scalar.

The energy momentum tensor (T_i^j) for the cloud of strings in the presence of perfect fluid is given by

$$T_i^j = (\rho + p) v_i v^j + p g_i^j - \lambda x_i x^j \tag{3}$$

Here ρ is proper energy density, p is pressure and λ is string tension density. Also x^i, the unit space like vector specifying the direction of strings and v^i, the unit time like vector satisfying the following conditions:

$$v_i v^i = -1 = -x_i x^i \; and \;\; v^i x_i = 0 \tag{4}$$

If the particle density of configuration is denoted by ρ_p, then it is given by

$$\rho = \rho_p + \lambda \tag{5}$$

The co-moving coordinate system is chosen as

$$v^i = (0,0,0,1); \quad x^i = \left(\frac{1}{A}, 0,0,0\right) \tag{6}$$

The Einstein's field equation (2) for metric (1) leads to

$$\frac{B_4^2}{B^2} + \frac{2B_{44}}{B} - \frac{\alpha^2}{A^2} + \Lambda = \lambda - p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\alpha^2}{A^2} + \Lambda = -p \tag{8}$$

$$\frac{B_4^2}{B^2} + \frac{2A_4 B_4}{AB} - \frac{3\alpha^2}{A^2} + \Lambda = \rho \tag{9}$$

$$2\alpha \left[\frac{B_4}{B} - \frac{A_4}{A}\right] = 0 \tag{10}$$

Solution of Field equations

The field equations (7) - (10) are the organization of four equations with six unidentified parameters A, B, λ, ρ, p and Λ, so to obtain the complete solution of the model we assume the matter to be stiff fluid,

i.e. $\rho - p = 0$ (11)

and Λ is inversely proportional to R^3, which leads to

$$\Lambda = \frac{a}{R^3} = \frac{a}{AB^2} \tag{12}$$

where a is constant.

Equation (10) leads to

$$\frac{A_4}{A} = \frac{B_4}{B} \tag{13}$$

On integrating equation (13), we get

$$A = Bk \tag{14}$$

where k is constant of integration.

Without loss of generality, we assume k=1, then equation (14) becomes

$$A = B \tag{15}$$

On adding equations (8) and (9), we get

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{3A_4B_4}{AB} + \frac{B_4^2}{B^2} - \frac{4\alpha^2}{A^2} + 2\Lambda = \rho - p \tag{16}$$

Using the equations (11), (12) and (15) in equation (16), we obtain

$$2A_{44} + \frac{4A_4^2}{A} = \frac{4\alpha^2}{A} - \frac{2a}{A^2} \tag{17}$$

Now, put $A_4 = f(A)$ and $A_{44} = ff'$ in equation (17), then

$$\frac{df^2}{dA} + \frac{4f^2}{A} = \frac{4\alpha^2}{A} - \frac{2a}{A^2} \tag{18}$$

On integrating equation (18), we obtain

$$f^2 = \alpha^2 - \frac{2a}{3A} + \frac{l}{A^4} \tag{19}$$

where l is integrating constant.

On integrating equation (19), we obtain

$$\int \frac{dA}{\sqrt{\alpha^2 - \frac{2a}{3A} + \frac{l}{A^4}}} = \int dt + M = t + M \tag{20}$$

where M is constant. Value of A can be obtained from equation (20).

After suitable transformation metric (1) reduces to the form

$$ds^2 = -\frac{dT^2}{\left[\alpha^2 - \frac{2a}{3T} + \frac{l}{T^4}\right]} + T^2dX^2 + T^2e^{2\alpha X}(dY^2 + dZ^2) \tag{21}$$

Here A=T, x=X, y=Y and z=Z.

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Physical and Geometrical Characteristics

Proper energy density (ρ) and pressure (p) for the model (21) are given by

$$\rho = p = \frac{3l}{T^6} - \frac{a}{T^3} \tag{22}$$

String tension density (λ) for the model (21) is given by

$$\lambda = \frac{4a}{T^3} \tag{23}$$

Particle density (ρ_p) for the model (21) is given by

$$\rho_p = \frac{3l}{T^6} - \frac{5a}{T^3} \tag{24}$$

Expansion (θ) and shear (σ) for the model (21) are given by

$$\theta = 3\left[\frac{\alpha^2}{T^2} - \frac{2a}{3T^3} + \frac{l}{T^6}\right]^{\frac{1}{2}} \tag{25}$$

$$\sigma = 0 \tag{26}$$

Hence, $\frac{\sigma}{\theta} = 0$ (27)

Magnitude of the rotation ω is identically zero.

Cosmological term Λ for the model (21) is given by

$$\Lambda = \frac{a}{T^3} \tag{28}$$

Conclusion

In the present paper, we obtained a spatially homogeneous and isotropic Bianchi-V string cosmological model for stiff perfect fluid distribution with cosmological term Λ.

The model (21) starts expanding with big-bang at T=0. The expansion (θ) is decreasing function of cosmic time T. Since, $\lim_{T\to\infty}\frac{\sigma}{\theta} = 0$, hence the model isotropized for large values of T. From equations (22) - (24) and equation (28), we can observe that the energy density (ρ), string tension density (λ), particle density (ρ_p), pressure (p) and cosmological term Λ are found to be a decreasing function of time T and approaches to 0 as T→∞.

Hence, in general, the present model represents expanding, non-shearing and non-rotating universe.

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